

# Binary Arithmetic Lab

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## Introduction:

We are used to representing numbers in the decimal system. The symbol 1 represents a single unit of something. The symbol 2 represents two units. The two symbols 10 placed together represent ten units, and so forth. However, this is just one way of representing numbers. We could also use tally marks, or Roman Numerals, or pictures.

Another alternative to the decimal system is the binary system. The decimal system uses base 10, meaning that we have 10 symbols to represent the numbers 0 - 9. To represent 10, we move the 1 symbol into another column, the tens column. These two columns are sufficient for numbers up to  $10^2$ , at which point we move the 1 symbol into another column, the hundreds column, to represent 100, and so forth.

In decimal, the symbols 123 give us the following information:

- There is a 1 in the hundreds ( $10^2$ ) column, so we have  $1 \cdot 10^2 = 100$
- There is a 2 in the tens ( $10^1$ ) column, so we have  $2 \cdot 10^1 = 20$
- There is a 3 in the units ( $10^0$ ) column, so we have  $3 \cdot 10^0 = 3$
- The symbols 123 thus represent the sum of  $100 + 20 + 3$

The binary system uses base 2. There are only 2 symbols now, enough to represent the numbers 0 and 1. To represent 2, we move the 1 symbol into another column, the twos column. 2 is represented as 10. These two columns are sufficient for numbers up to  $2^2$ , at which point we move the 1 symbol into another column, the fours column, to represent 4, which looks like 100.

In binary, the symbols 101 give us the following information:

- There is a 1 in the fours ( $2^2$ ) column, so we have  $1 \cdot 2^2 = 4$
- There is a 0 in the twos ( $2^1$ ) column
- There is a 1 in the ones ( $2^0$ ) column, so we have  $1 \cdot 2^0 = 1$
- The symbols 101 thus represent the sum of  $4 + 1$

To convert from binary to decimal, we can use the algorithm from the above example. Multiply each digit, a 1 or a 0, by the appropriate power of 2, depending on its column, and add the results.

To convert from decimal to binary, determine the greatest power of 2 that will fit into the decimal number. Write a 1 in the column corresponding to that power of 2. Then subtract that power of 2 from the decimal number. Consider the next lowest power of 2; does that fit into the remainder? If so, write a 1 in the next column, and subtract that power of 2 from the remainder. If not, write a 0 in the next column. Now consider the next lowest power of 2, and continue the process until you reach 0.

We'll call the above conversion methods the "powers of 2" methods. It happens that there are, in fact, easier methods, which we'll call the "doubling" and "halving" methods.

We can use an example to illustrate converting from binary to decimal using the "doubling" method. We'll convert the binary representation 11010. Starting with 0, double your number ( $2 \cdot 0 = 0$ ) and add a digit from the binary representation (we begin at the leftmost digit, which is always 1, so  $0 + 1 = 1$ ). Now move to the next digit. Double your number ( $2 \cdot 1 = 2$ ) and add the digit from the binary representation ( $2 + 1 = 3$ ). The rest of the process is presented in the table below:

<u>1</u> 1 0 1 0	$2 \cdot 0 + 1 = 1$
1 <u>1</u> 0 1 0	$2 \cdot 1 + 1 = 3$
1 1 <u>0</u> 1 0	$2 \cdot 3 + 0 = 6$
1 1 0 <u>1</u> 0	$2 \cdot 6 + 1 = 13$
1 1 0 1 <u>0</u>	$2 \cdot 13 + 0 = 26$

The final answer is 26.

Again, we can use an example to illustrate converting from decimal to binary using the "halving" method, which is essentially the reverse of the above method. We'll convert the decimal representation 46. First we divide by 2, and note the remainder.  $46/2$  is 23, so the remainder is 0. This remainder will be the rightmost digit of our binary representation. Next we divide the quotient by 2 and place the remainder in the leftmost slot of our binary representation. Since  $23/2$  is 11 with a remainder of 1, we put a 1 next to the 0, so that so far our binary representation is 10. The next number we divide by 2 is 11. The rest of the process is presented in the table below:

Division	Remainder	Binary Representation
$46 / 2 = 23$	0	0
$23 / 2 = 11$	1	1 0
$11 / 2 = 5$	1	1 1 0
$5 / 2 = 2$	1	1 1 1 0
$2 / 2 = 1$	0	0 1 1 1 0
$1 / 2 = 0$	1	1 0 1 1 1 0

The final answer is 101110.

Doing arithmetic in binary is analogous to doing arithmetic in decimal. Adding two single-digit binary numbers is very simple:

$$0 + 0 \rightarrow 0$$

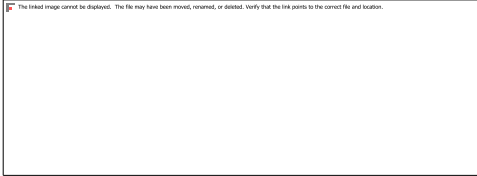
$$0 + 1 \rightarrow 1$$

$$1 + 0 \rightarrow 1$$

$$1 + 1 \rightarrow 10 \text{ (or 0, carry 1)}$$

Adding  $1 + 1$  results in 0 with a 1 carried to the next column. That's just like what happens in decimal when we add, for example,  $5 + 6$  -- we get a 1 in the ones place, with a 1 carried into the next column, in the tens place.

We add multiple-digit binary numbers digit-by-digit, moving from right to left, as we do in binary. In the example below, we add 1101 to 10111 (the carried digits are stored in the top row):



**Problem Set:**

1. Convert the following binary numbers into decimal by expanding into a sum of powers of 2.

Example:  $101 = 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 2^2 + 2^0 = 4 + 1 = 5$

- A. 100
  - B. 1010
  - C. 101110
  - D. 1100011
2. Convert the following decimal numbers into binary by subtracting powers of 2.
    - a. 14
    - b. 25
    - c. 48
    - d. 91

3. Convert the following binary numbers into decimal using the "doubling" method. Show your work in steps, and for each step, indicate the relevant digit of the binary representation and the corresponding arithmetic.

Example: 1011

<u>1</u> 0 1 1	$2 \cdot 0 + 1 = 1$
1 <u>0</u> 1 1	$2 \cdot 1 + 0 = 2$
1 0 <u>1</u> 1	$2 \cdot 2 + 1 = 5$
1 0 1 <u>1</u>	$2 \cdot 5 + 1 = 11$

a. 1101

b. 10111

c. 11001

d. 101001

4. Convert the following decimal numbers into binary using the "halving" method. Show your work in steps, and for each step, indicate the division performed, the remainder, and the binary representation so far.

Example: 29

Division	Remainder	Binary Representation
$29 / 2 = 14$	1	1
$14 / 2 = 7$	0	0 1
$7 / 2 = 3$	1	1 0 1
$3 / 2 = 1$	1	1 1 0 1
$1 / 2 = 0$	1	1 1 1 0 1

a. 17

b. 36

c. 53

d. 69

5. Add the following one-bit binary numbers, indicating the sum and carry in your answer:

a.  $1 + 0$

b.  $1 + 1$

c.  $1 + 1 + 1$

6. Add the following multi-bit binary numbers:

a.  $1010 + 100$

b.  $10 + 11$

c.  $1001 + 1011$

d.  $11011 + 1010$