Numeration

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Symbols

- Symbols are a means of communicating facts and ideas:

  I have three cows and two sheep

  I will see you tomorrow

- Clay tablets in Sumer were used for pictographic writing @ 3300 BC
- Egyptians use hieroglyphic signs on papyrus

Clay Tablet

Symbols:

- **English: 4 7 A a Z z**
- **International: Ê Š Æ Ü ç ê ñ**
- **Mathematics: + - / * ^ ƒ €**
- **Special characters: ♋ @ ® ™ © ♣ 😐**
- **Greek: Φ Γ Π Σ θ ω Δ**

Numeration

- Virtually all numeration starts as tallies, using single strokes to represent each additional unit: / for one, // for two, etc.
- Evidence of tallies has been found on *bone fragments* from as early as 15,000 BC.
- A tally system can exist before a language develops words for numbers.
- Reference: Bunch and Hellemans, *The Timetables of Technology*, Simon & Schuster, 1993

Tally Stick

Tokens

- Early societies developed tokens to represent quantities.
- By 4000 BC, tokens existed for "ten sheep" (say: ) and for "one sheep"(say: ●)

- Given the following tokens: 
  - *How many sheep are represented?*

- There were different tokens for different commodities!

  Three horses would be represented as "▲ ▲ ▲" and not "● ● ●," which is three sheep!

Concept of *Number*

- Around 4000 BC, traders in Uruk were discovering that the same number could be used to mean ten sheep, ten bags of grain, or ten talents of copper.

- About 3000 BC, Egyptian tallies show items grouped at ten;
  - these tallies were regrouped at a hundred,
  - and regrouped again at one thousand.

Hieroglyphic numbers

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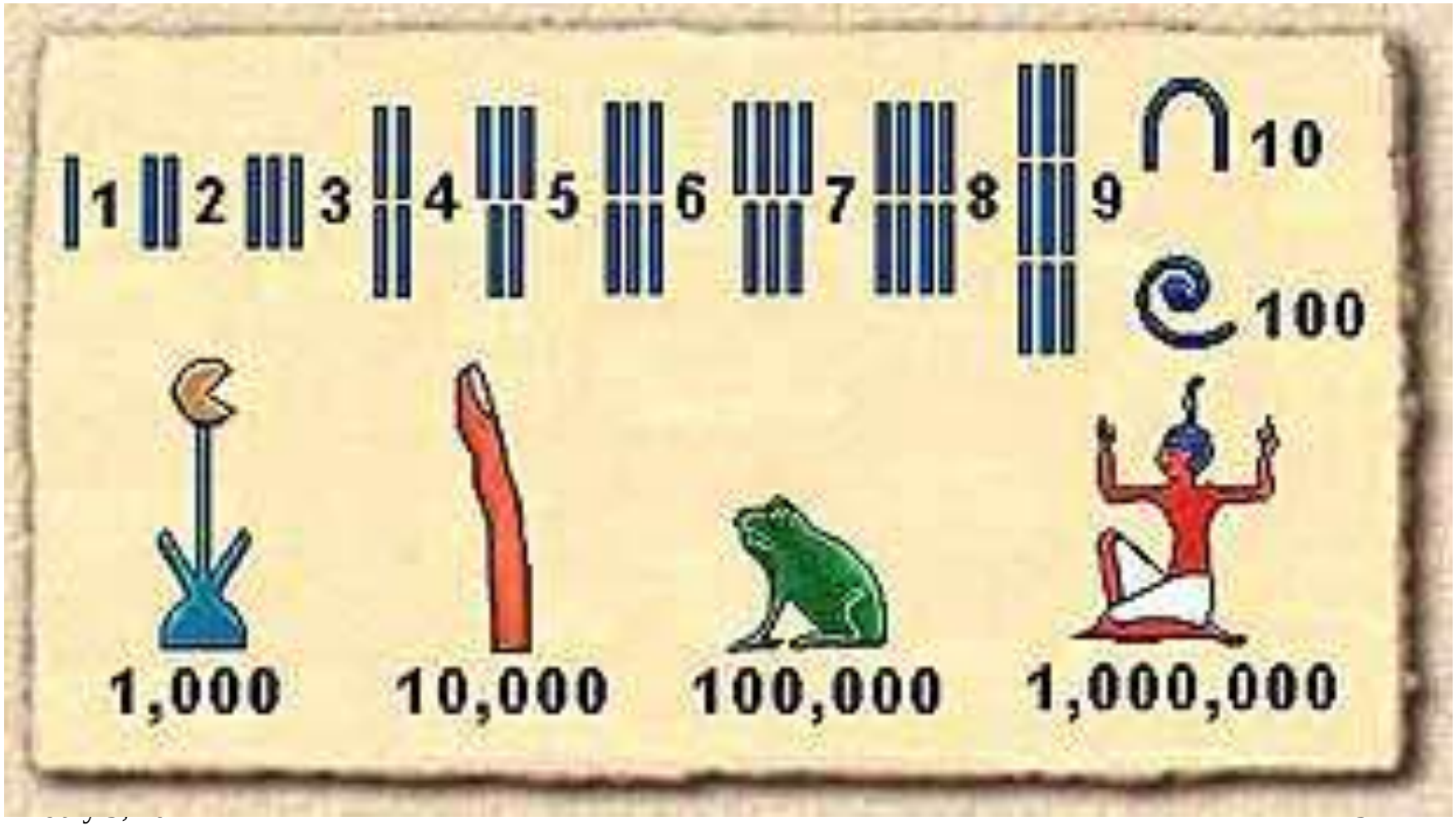

Two examples

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12,425 Birds

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Egyptians

Hieroglyphics (pictographic symbols)

- 1 = Stick /
- 10 = Arch Π
- 100 = Coiled Rope ℘
- 1000 = Lotus Flower ✶
- 10,000 = Finger (pointing to sky) (
- 100,000 = Tadpole (from the Nile)
- 1,000,000 = Man (arms reaching to heaven)

Hieroglyphics **(addition)**

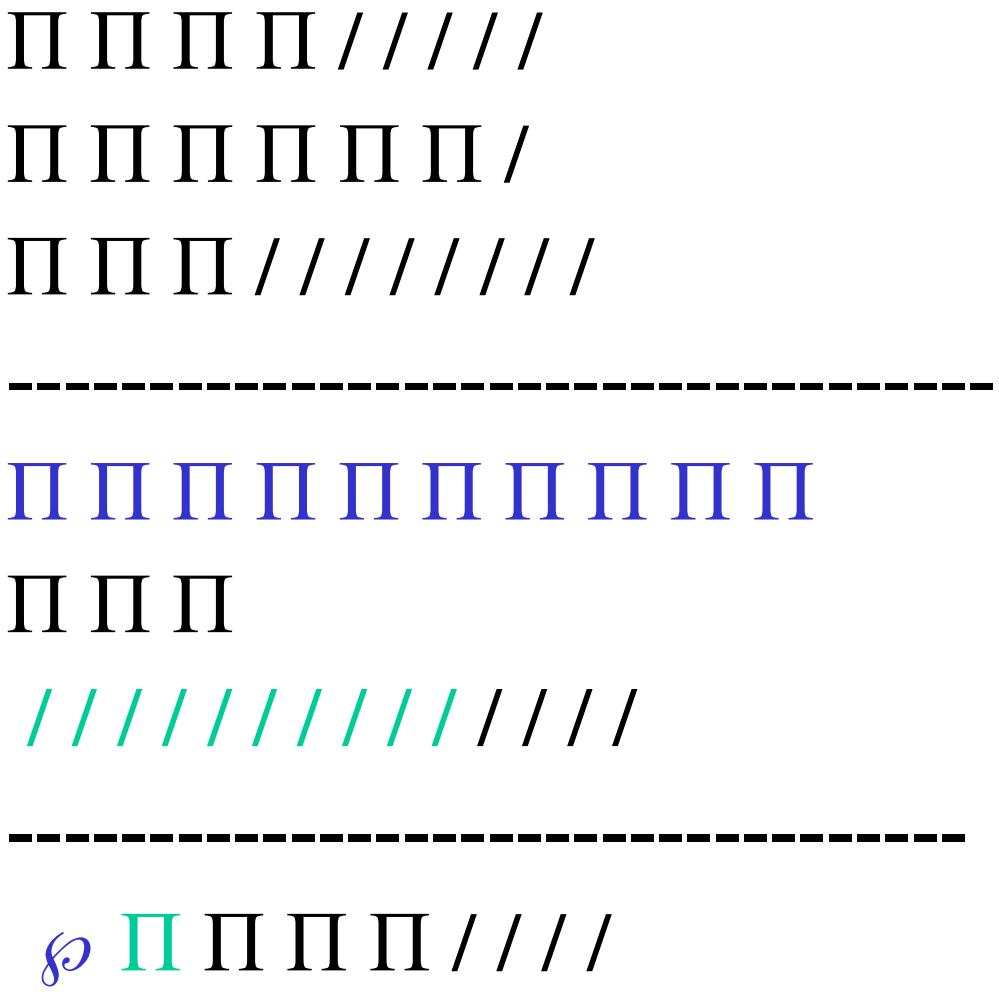
- Π Π Π Π / / / / /
- Π Π Π Π Π Π /
- Π Π Π / / / / / / /

45

61

38

Hieroglyphics **(addition)**



Multiplication by Doubling (23 X 13)

Number	*multiplier*
Π Π / / /	**1**
Π Π Π Π / / / / / /	**2**
Π Π Π Π Π Π Π	**4**
Π Π / /	
℘ Π Π Π Π Π Π Π	**8**
Π / / / /	

Number	*multiplier*
23	**1**
46	**2**
92	**4**
184	**8**

Multiplication by Doubling (23 X 13)

Number	*multiplier*
Π Π / / /	**1** ♋
Π Π Π Π / / / / / /	**2**
Π Π Π Π Π Π Π Π	**4** ♋
Π Π / /	
℘ Π Π Π Π Π Π Π Π	**8** ♋
Π / / / /	

Number	*multiplier*
23	**1** ♋
46	**2**
92	**4** ♋
184	**8** ♋
==========	
299	**13**

Check and verify!!!

$$
\begin{array}{r}
23 \\
\times\ 13 \\
\hline
69 \\
23\_ \\
\hline
299
\end{array}
$$

Greeks and Romans

- The Greeks adapted their alphabet for numerals; others followed their example.
- Roman numerals are also alphabetical, but they did not originate as such. Early artifacts show that the X for ten, originated from the way in which scribes drew a slanted line through the number for four:
- ///// + / became X; one half of X was V, and the habit of putting a circle around the tenth X to indicate one hundred became C

Roman Numerals

I	**XI**	**XXX**	**XXXX**	**L**
II	**XII**			
III	**XIII**			
IV	**XIV**			
V	**XV**			
VI	**XVI**			
VII	**XVII**			
VIII	**XVIII**			
IX	**XIX**			
X	**XX**			

Roman Numerals

L = 50

C = 100

D = 500

M = 1000

V (bar)= 5000 "vee bar"

XV (bar) = 15,000

L (bar) = 50,000

C (bar) = 100,000

M (bar)= 1,000,000

Addition using Roman Numerals

2318	**MM CCC X V III**
+821	**DCCC XX I**
3139	**MM D CCC XXX V IIII**
	CCC

	MM IX
	D
collecting	**D D C XXX**
terms	**----------**
	MMM C XXX IX

Subtraction using Roman Numerals

2486		**MM CCCC L XXX V I**
-1343		**M CCC XXXX III**
1143	**expand:**	***MM CCCCXXXXX IIIII XXX***
|	**minus:**	***M CCC XXXX III***
|		
|----------→		**M C XXXX III**

Multiplication using Roman Numerals

		I	*V*	*X*	*L*	*C*	*D*	*M*	
V	|	*V*	*XXV*	*L*	*CCL*	*D*	*MMD*	*V-bar*	
X	|	*X*	*L*	*C*	*D*	*M*	*V-bar*	*VV-bar*	

2 8 **XXVIII**

times 1 2

5 6 **XXVIII times 1 = XXVIII**

2 8 **XXVIII times 1 = XXVIII**

3 3 6 **XXVIII times 10 = CCLXXX**

CC L XXXXXXX VV IIIIII

Collect terms: ***CCC XXX VI***

Hindu-Arabic notation

- The Indians used horizontal tallies (/) for one, two and three, and special symbols for four through nine.
- Around 600 CE, the Indians started using place values, i.e., instead of writing the equivalent of 100 + 80 + 7, they wrote 187
- Only nine digits were used along with a symbol for zero, probably derived from astronomer's marking empty places.
- *A famous inscription dated 870 CE contains the first zero that has survived.*

Hindu-Arabic numerals

- **Ancient Hindus:**
  - **zero**
  - **place values and decimal system (base 10)**
  - **Positional Notation:** **4, 4 2 8**

- **Arab traders brought the system to Europe where it became known as "Arabic numerals"**
-
- **base:** X^5 X^4 X^3 X^2 X^1 X^0
- **10** **100000** **10000** **1000** **100** **10** **1**
- **2** **32** **16** **8** **4** **2** **1**

House of Wisdom

- **Caliph Al-Mamun**
  - **800 AD**
  - **Baghdad**

- **Prophet Mohammed: “Seek Learning Though It Be In China”**
  - **Astronomical tables - Feast of Ramadan**
  - **Mecca (geography and geometry)**

Al-Khowarizmi

Abu Jafar Mohammed Ibn Musa Al-Khowarizmi

born 780 AD

- **Kiva, USSR**
- ***Hisab Al Jabr Wal-Mugabalah*** **(The Compendious Book on Calculations by Completion And Balancing)**
- **Used Hindu numerals and decimal system**
- **Spread throughout Europe**
- **“business” problems: inheritance of estates**
- **modern words: algorithm from “Al-Khowarizmi”**
- **and algebra from “Al Jabr”**

– **wrote 2 additional books on the Astrolabe**

Mesopotamians

- Number system based on sixty: 60
- Through the ages this system has been used by astronomers:

  60 seconds in a minute

  60 minutes in an hour

  360 degrees in a circle

  longitude and latitude

Early European Textbooks

Hero of Alexandria: multiplication by factoring

13 times 8 = (10 + 3) times (10 + 8) =

100 + 80 + 30 + 24 = 234

Ptolemy (The Almagest)

Used Babylonian number system

Base 60, our source of 60 minutes, 60 seconds, and 360º angular measurements

Mathematical Operations

Addition, Subtraction, Multiplication, Division,

Duplation (doubling), and Mediation (halving)

Multiplication by doubling

297

times 22

5 9 4

5 9 4

6 5 3 4

297	**1**
594	**2** ♋
1188	**4** ♋
2376	**8**
4752	**16** ♋
6534	**22**

The Instruction of Ptahotep:

No limit may be set to art, neither is there any craftsman that is fully master of his craft.

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Alphabetic symbols

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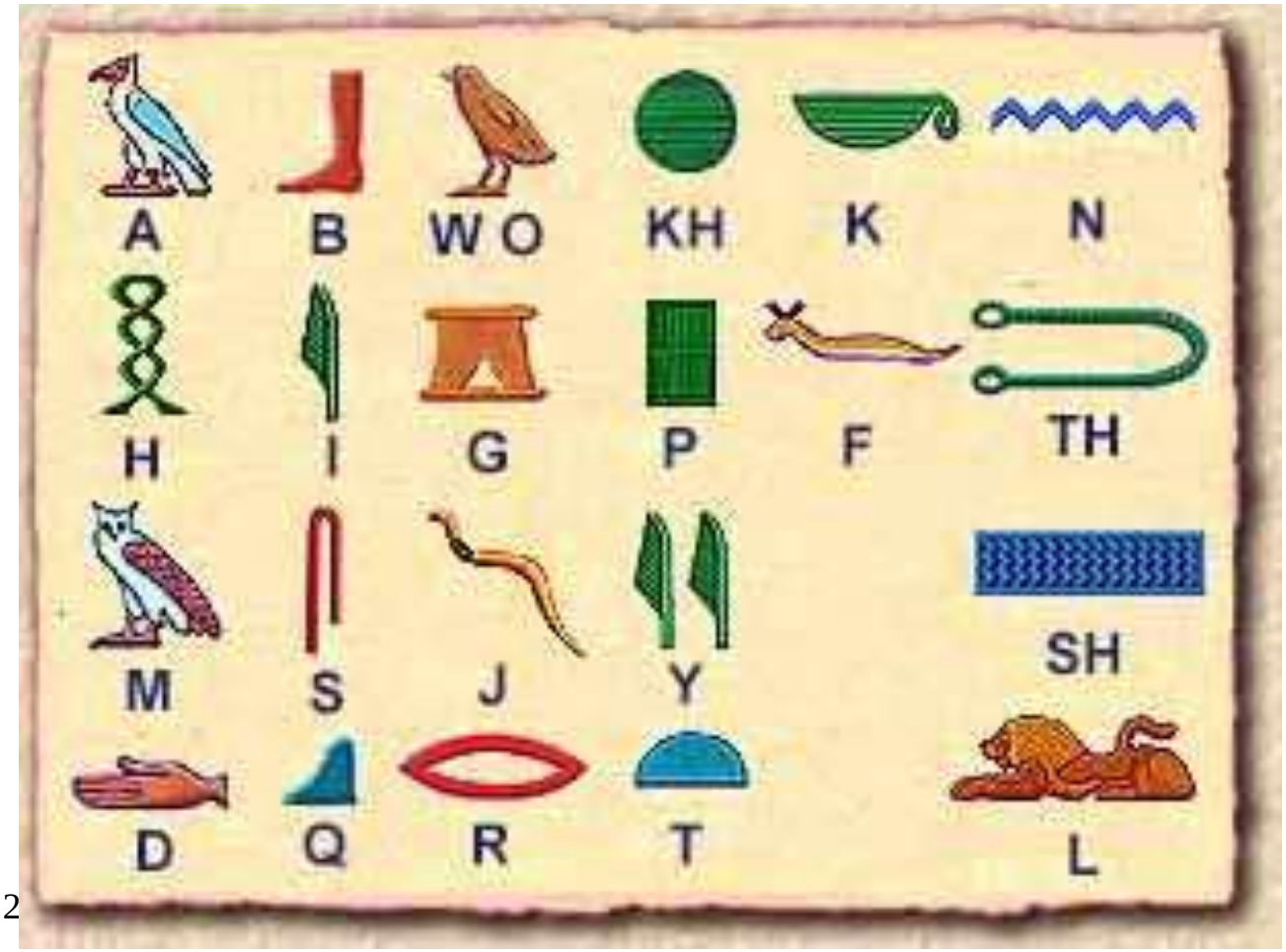

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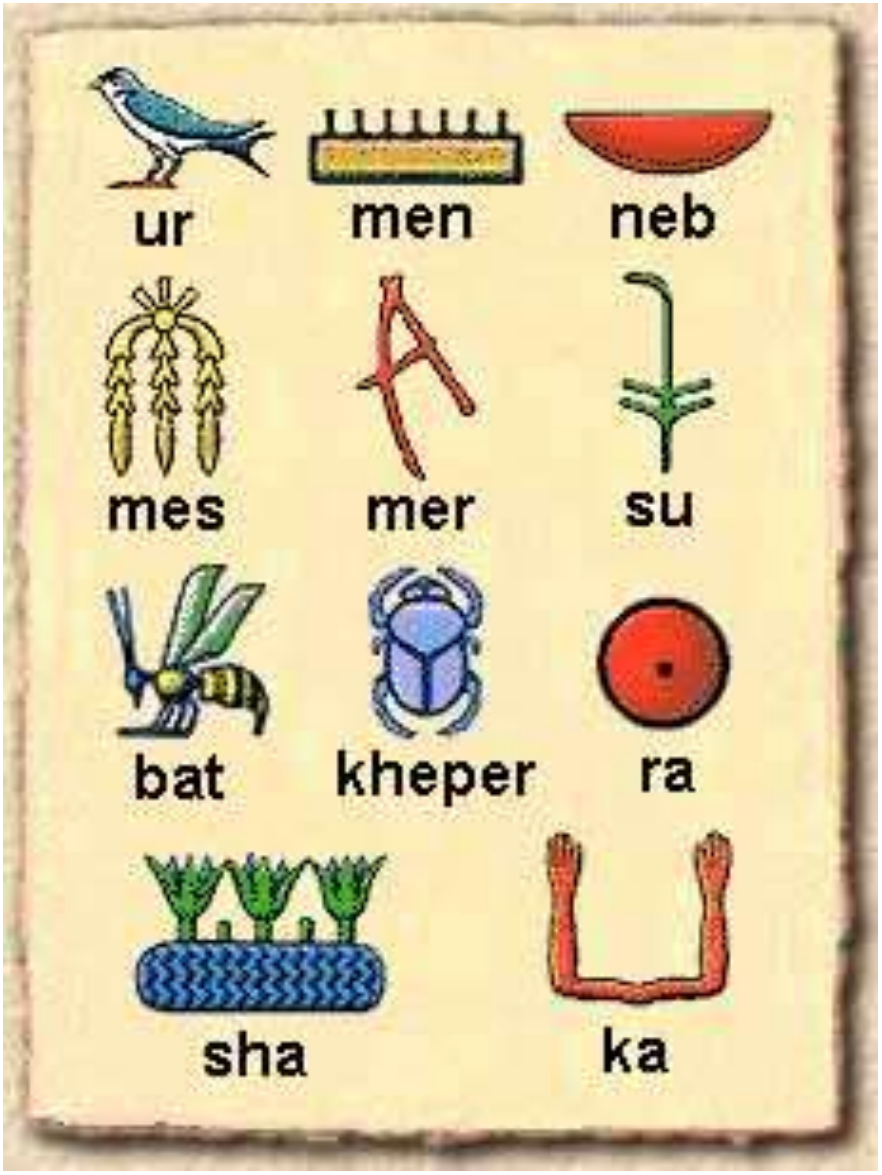

Alphabets

- 1500 BC one of the first alphabets is developed in Ugarit (Syria) by stripping down Mesopotamian cuneiform characters to only 30 signs; elsewhere in the middle east, scribes developed symbol sets that were easier to write than the wedge-shaped letters of cuneiform.
- 1000 BC Phoenicians develop an alphabet of 22 signs for consonants; although not the first alphabet, it is adapted by both Greeks and Israelites to their own needs.

Spread of Alphabets

- Because the Phoenicians were great traders they spread their version of the alphabet around the Mediterranean: Greeks, Etruscans, and Romans.
- After printing was invented, a form of the Roman alphabet from Italy became the standard printed alphabet. [Johann Gutenberg invented a system for casting type as a flat surface around 1440 CE]
  - Note how the English language, Windows®, and HTML have permeated present societies around the world.

References

- Bunch and Hellemans, *The Timetables of Technology*, Simon & Schuster, 1993

- A major source of information about hieroglyphics can be found at:

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Show and Tell

- Sample hieroglyphic tablets
- Prayer rug
- Astrolabe; hourglass