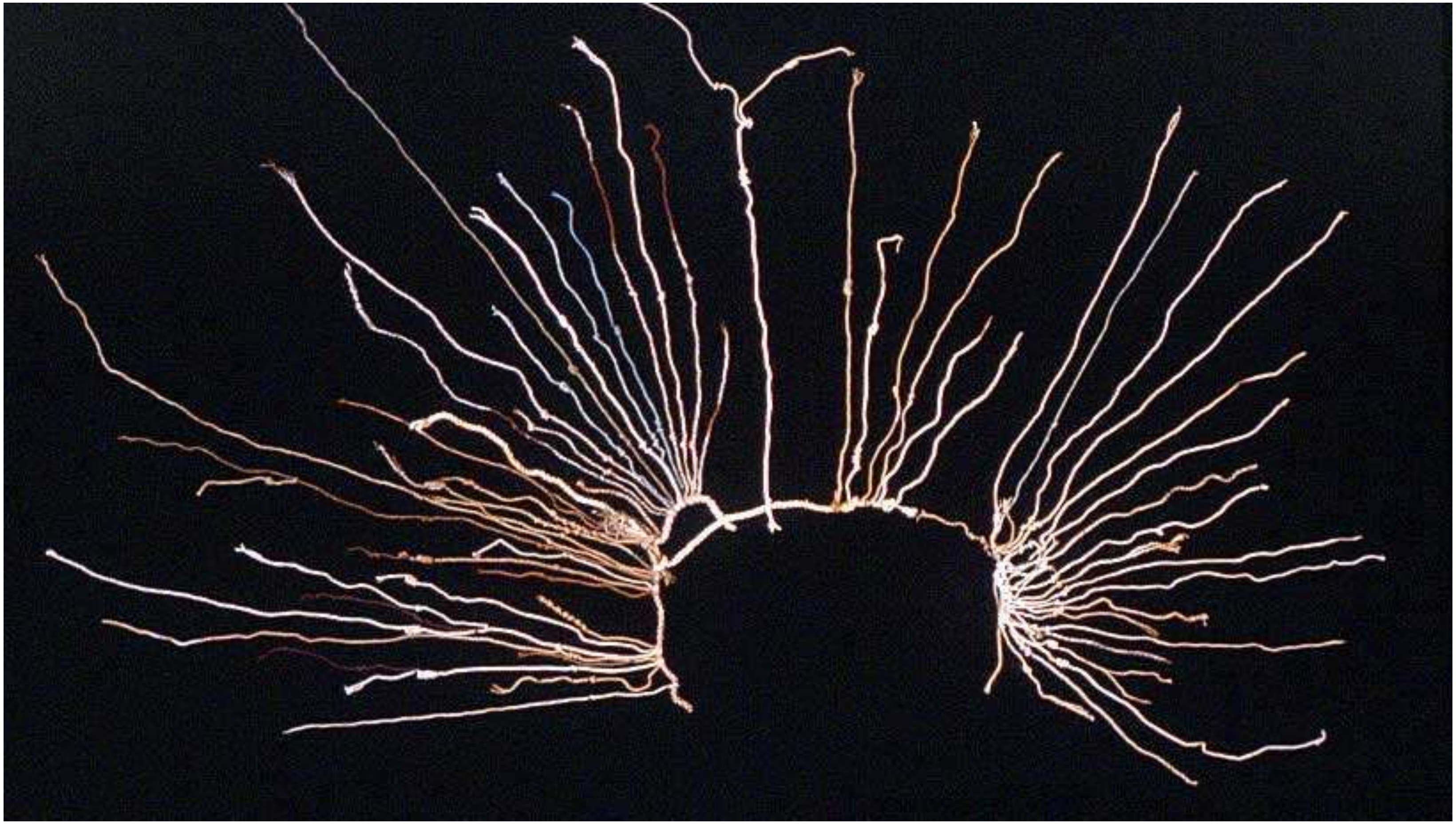
Early Aids To Calculation

Thomas J. Bergin

American University

Quipu

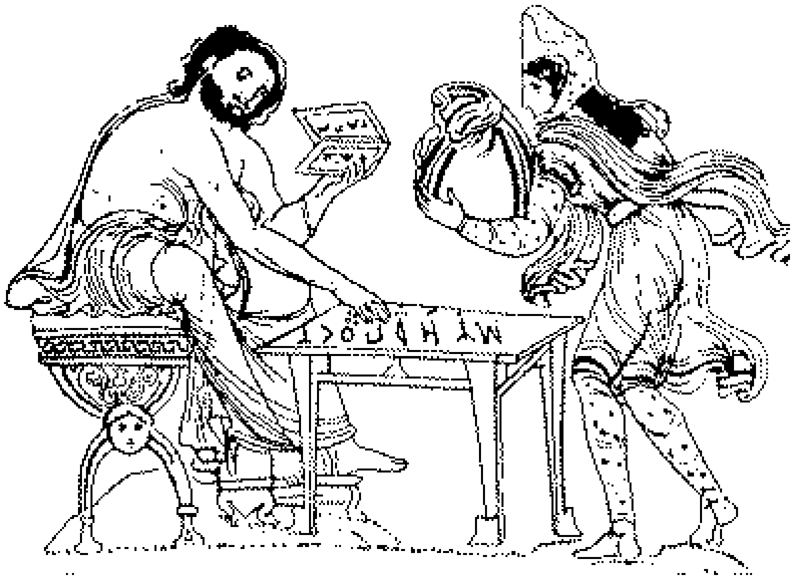


Pebbles

- -Calculus: n; pl. calculi, calculuses
  - (Latin, calculus, a small stone or pebble used in reckoning; diminutive of calx, calcis -- a stone)
  - 1. A small stone or pebble
  - 2. Any hard, solid concentration or deposit or any part of the body…
  - 3. In higher mathematics, (a) a method of calculation; (b) the use of symbols: (c) a method of analysis;…
    - --*Webster's New Twentieth Century Unabridged Dictionary*

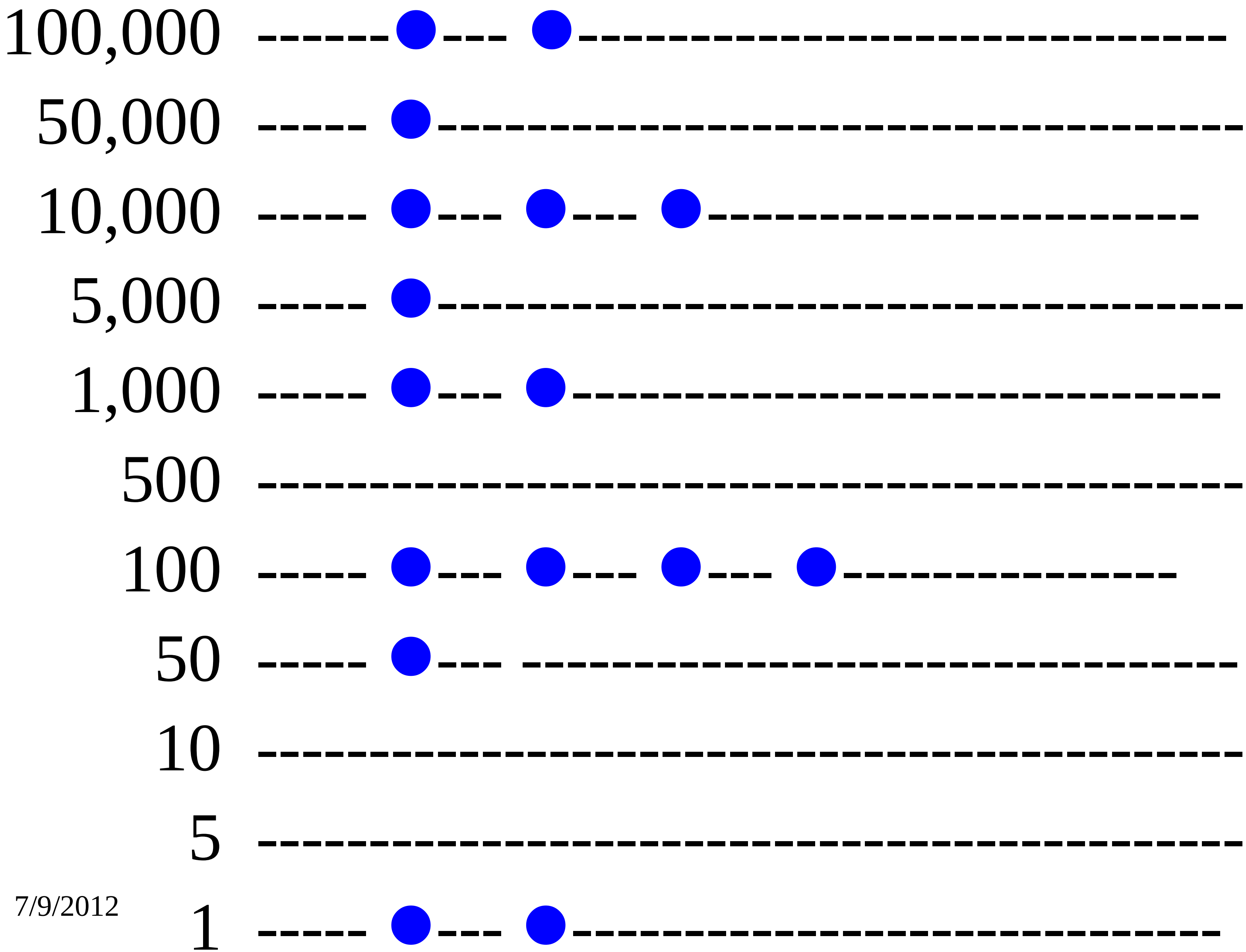
- Calculus of Finite Differences
- Calculus of Functions
- Calculus of Imaginaries
- Calculus of Operations
- Calculus of Probability
- Calculus of Variations
- Differential Calculus
- Integral Calculus

Abacus



- 3000 BCE, early form of beads on wires, used in China
- From semitic *abaq*, meaning dust.

Table Abacus

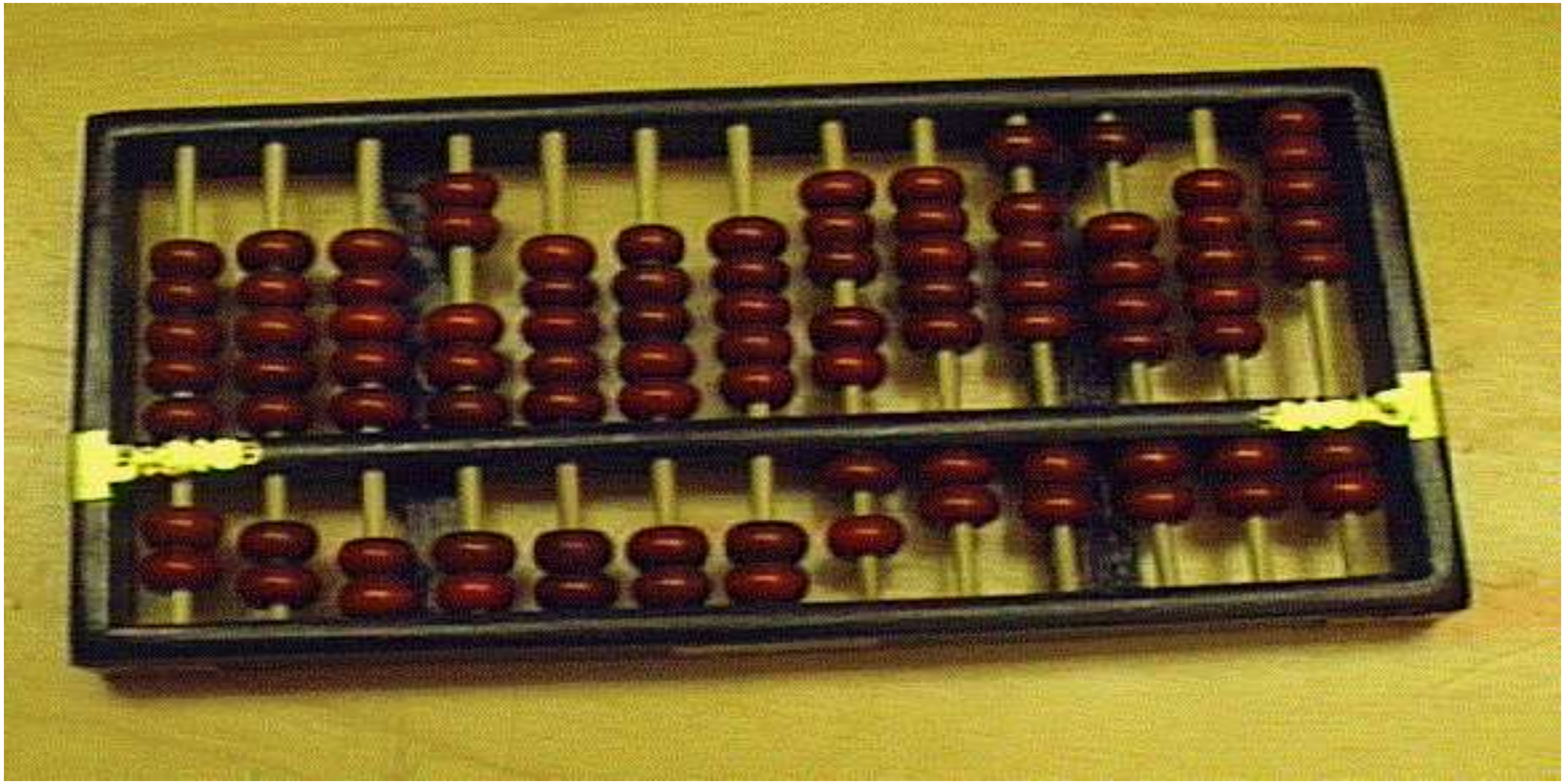


The "Office"

- Medieval counting boards with minted counters

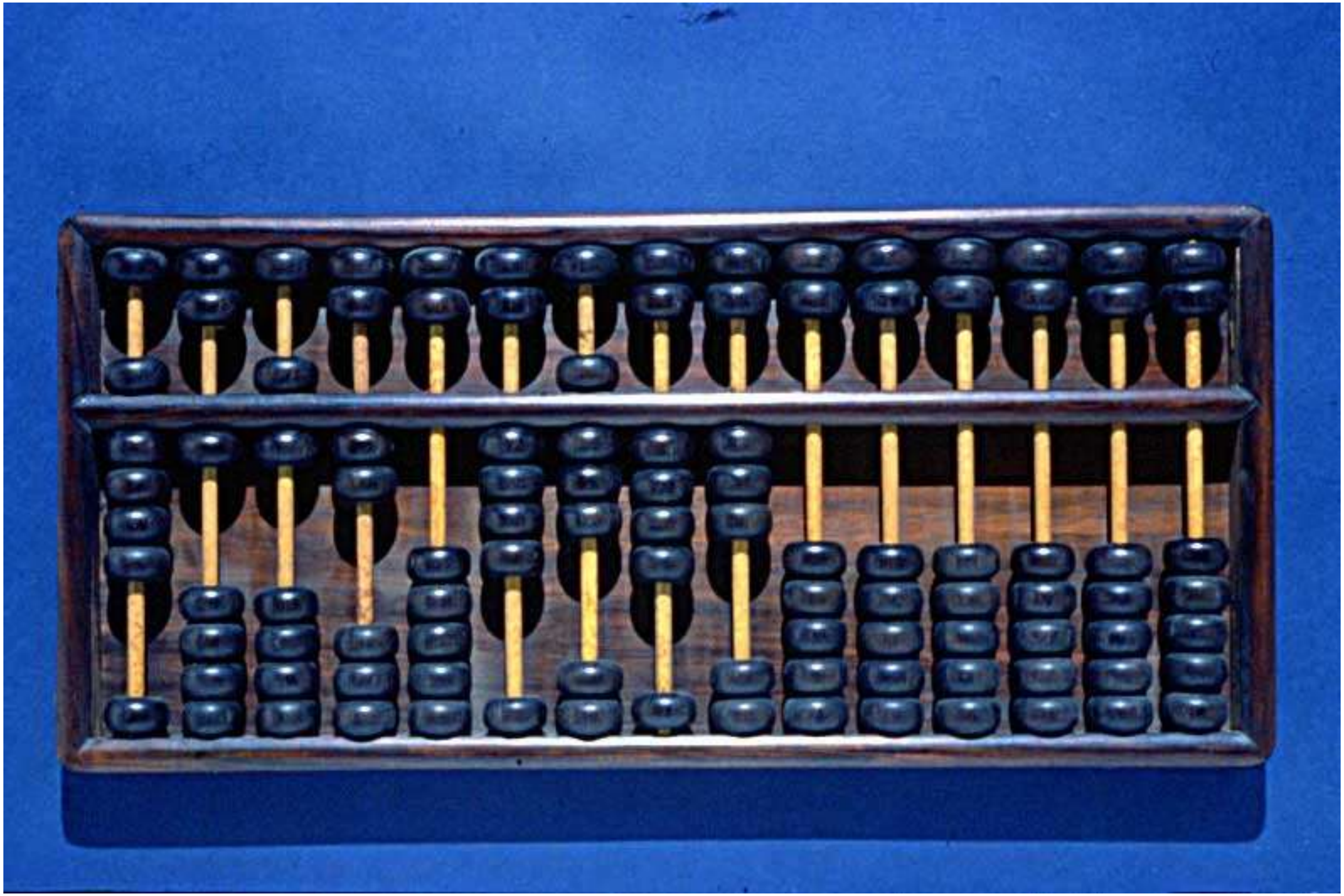
  (c 1200 Italy)
- Origin of:
  - "a counting"
  - to cast up an account
  - to cast a horoscope
  - *jetons*, *from the* French *jeter*, *verb*: to throw

Chinese Swan Pan

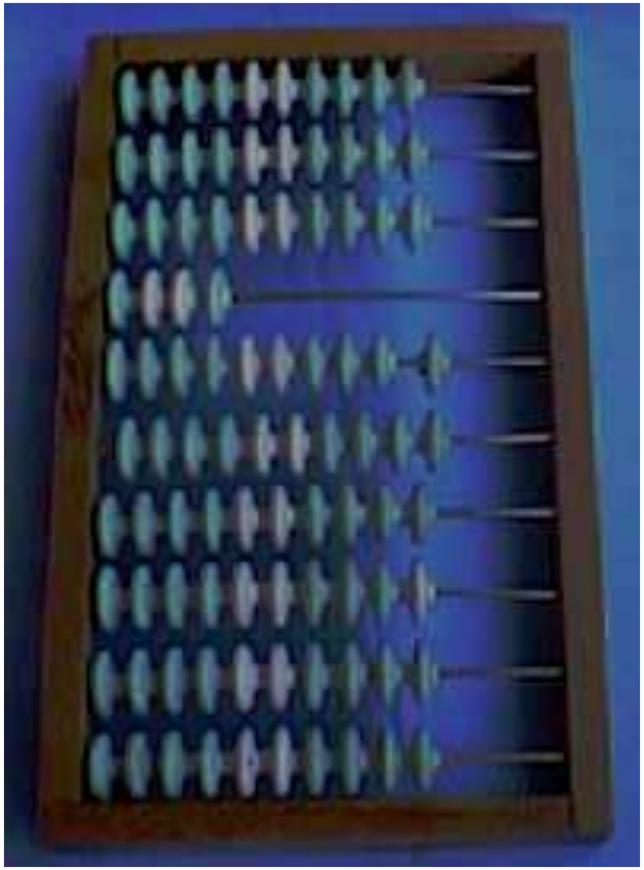


Exercise 1: Using an Abacus

- Using an abacus calculate:
- 12 + 15
- 35 - 22
- 24 X 33
- 89 / 12



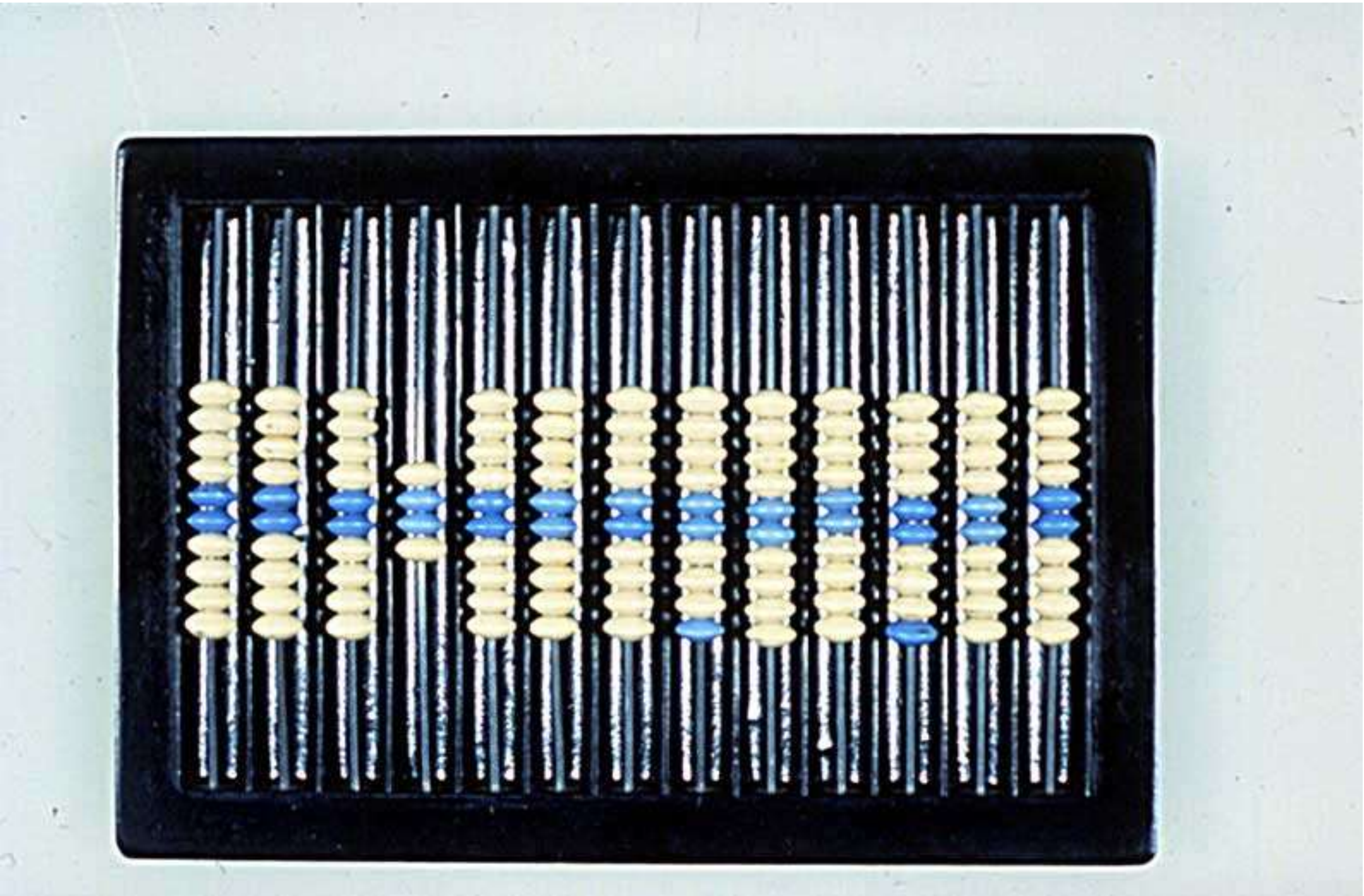
Russian Abacus



- Essentially a counting device
- Still in use in Russia and environs today!

Japanese Soroban

Indian bead abacus



Finger Reckoning

- “Educated” people memorized up to 5 times 5

- Assume fingers = 5 + number extended:

  | | is seven (five plus two)

  | | | is eight (five plus three)

- Add the *extended* fingers and multiply the *folded* fingers!

Finger Reckoning

- 0 to 5 — *memorized*
- 6 to 10 — $10(e + e') + cc'$
- 11 to 15 — $10(e + e') + cc' + 100$
- *16 to 20* — $20(e + e') + cc' + 200$
- *21 to 25* — $20(e + e') + cc' + 400$
- *26 to 30* — $30(e + e') + cc' + 600$

Exercise 2: Finger Reckoning

- Multiply:
  - 9 times 9
  - 8 times 8
  - 7 times 7
  - 6 times 6
  - 6 times 5

John Napier (1550-1617)

- Napier's ***Cannon of Logarithms*** 1614
  - Numbers in an arithmetic series are the logarithms of other numbers in a geometric series, to a suitable base.

- Napier's ***Rabdologia*** 1617
  - aka **Napier's "Bones"**
  - *Multiplicationis Promptuarium*

Mirifici Logarithorum Canonis Descriptio

Description of the Admirable Cannon of Logarithms

Natural numbers: 1 2 4 8 16 32 64 128

a geometric series: previous times 2

Base 2 logarithms: 0 1 2 3 4 5 6 7

arithmetic series: $2^0 = 1$, $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, *etc.*

Multiplication:

8 times 16 = ????

Log(8) + log(16) =

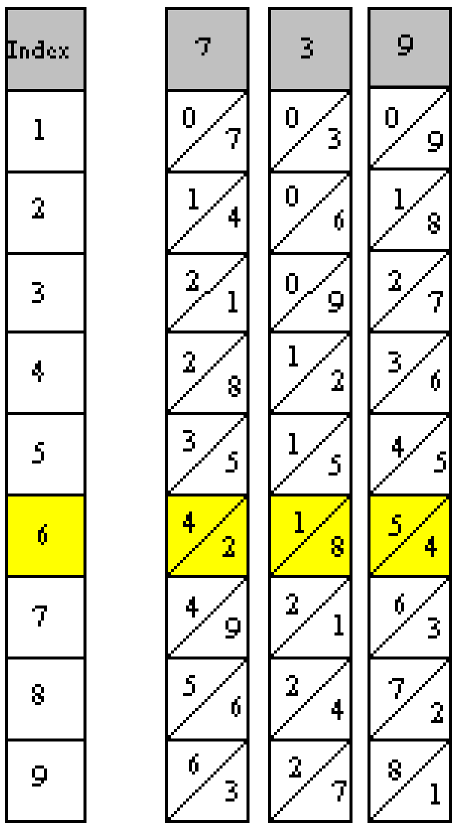
3 + 4 = 7 (the antilog of 7 is 128)

Exercise 3: Using Log Tables

- Examine logarithm tables
- Multiply two numbers using logarithms

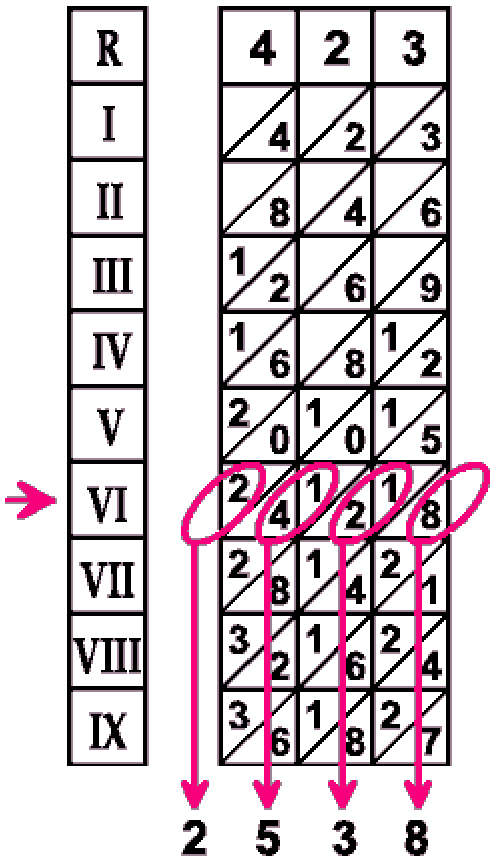
Napier's ***Rabdologia (1617)***

Napier's "Bones"



- Made out of animal bones (ivory)
- About the size of a cigarette and in a leather pouch
- Multiples, multiples, multiples!
- Arrange multiplicand and read multiplier row

Napier's "Bones"



- Multiplicand is: 423
- Align bones for 4, 2 & 3
- Add values on the diagonal
- Be careful of carries!
- Write down resultant

Partial Products

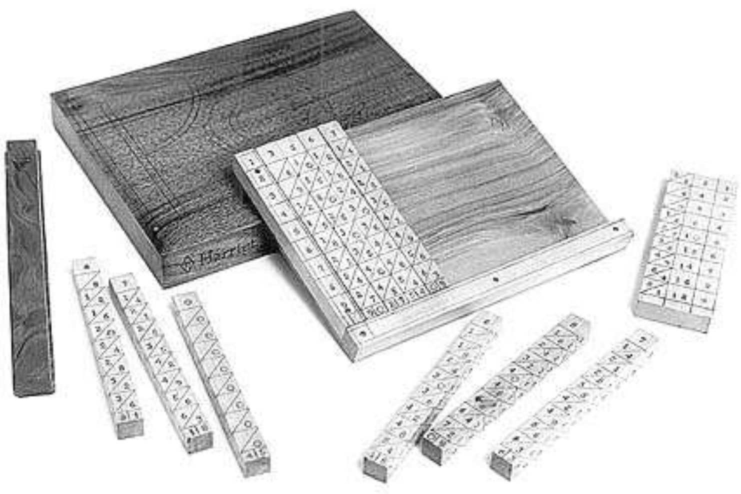
0	1	2	3	4	5	6	7	8	9	
0/0	0/1	0/2	0/3	0/4	0/5	0/6	0/7	0/8	0/9	1
0/0	0/2	0/4	0/6	0/8	1/0	1/2	1/4	1/6	1/8	2
0/0	0/3	0/6	0/9	1/2	1/5	1/8	2/1	2/4	2/7	3
0/0	0/4	0/8	1/2	1/6	2/0	2/4	2/8	3/2	3/6	4
0/0	0/5	1/0	1/5	2/0	2/5	3/0	3/5	4/0	4/5	5
0/0	0/6	1/2	1/8	2/4	3/0	3/6	4/2	4/8	5/4	6
0/0	0/7	1/4	2/1	2/8	3/5	4/2	4/9	5/6	6/3	7
0/0	0/8	1/6	2/4	3/2	4/0	4/8	5/6	6/4	7/2	8
0/0	0/9	1/8	2/7	3/6	4/5	5/4	6/3	7/2	8/1	9

Square and Cube Roots

$\sqrt{}$		
0/1	2	1
0/4	4	2
0/9	6	3
1/6	8	4
2/5	10	5
3/6	12	6
4/9	14	7
6/4	16	8
8/1	18	9

$\sqrt[3]{}$			
0/0	1	1	1
0/0	8	4	2
0/2	7	9	3
0/6	4	16	4
1/2	5	25	5
2/1	6	36	6
3/4	3	49	7
5/1	2	64	8
7/2	9	81	9

Exercise 4: Gelosia Method



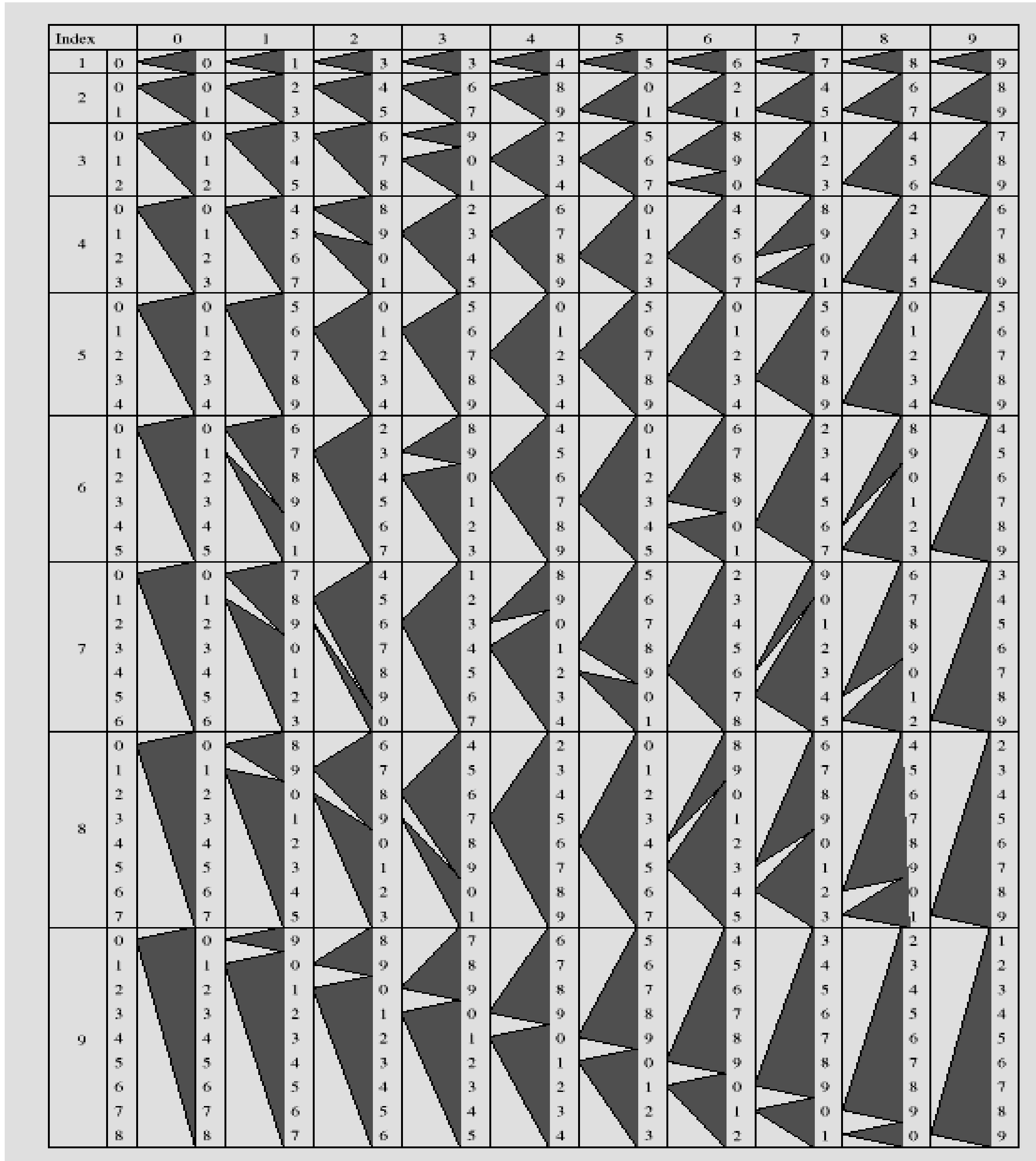
- Gelosia method:

  7843

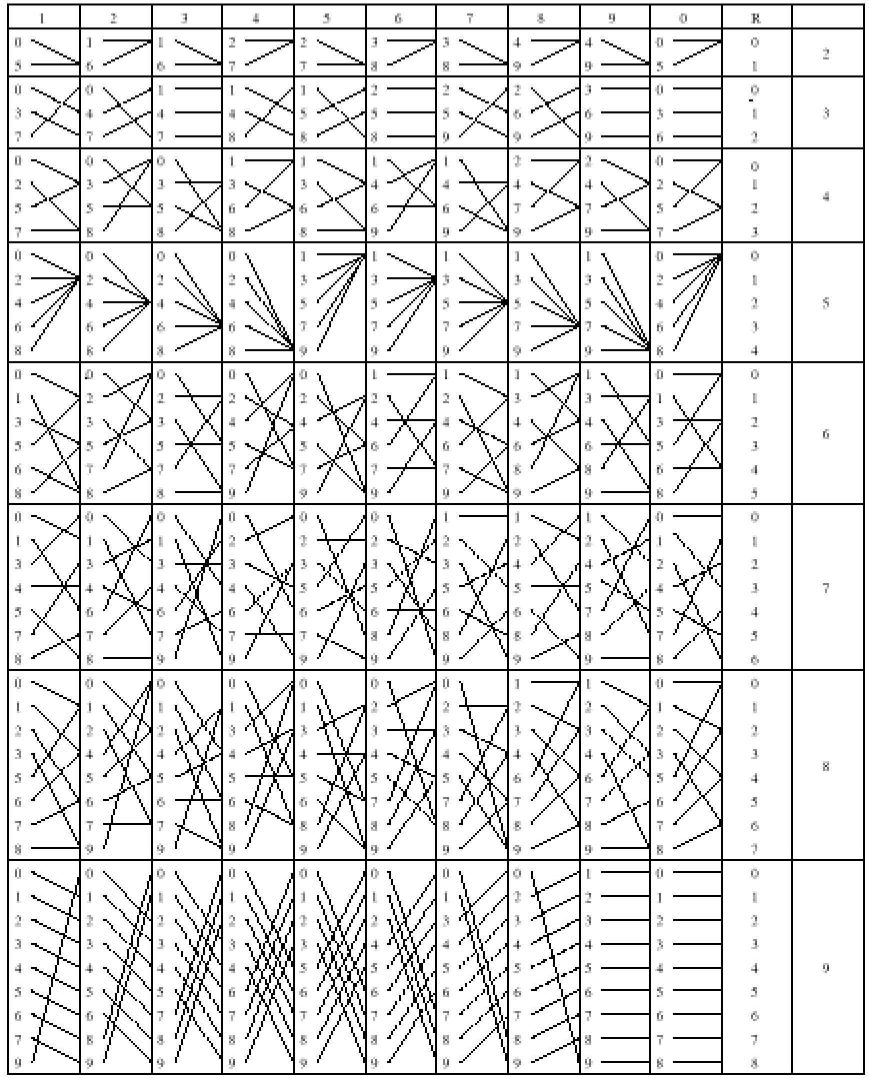
  X 9625

- Try again with someone doing it on the blackboard!

Genaille-Lucas Rulers (1885)



Genaille-Lucas Division Rulers



Slide Rule

- Napier's Logarithms
- Edmund Gunter: *Gunter's Line of Numbers (1620)*
- William Oughtred (left)
- (1574? to 1660)
- invented the slide rule by putting *Line of Numbers* on top of each other
- *Circles of Proportion*, as circular slide rules (1622)

Logarithms and the Slide Rule

Base 10 log of:

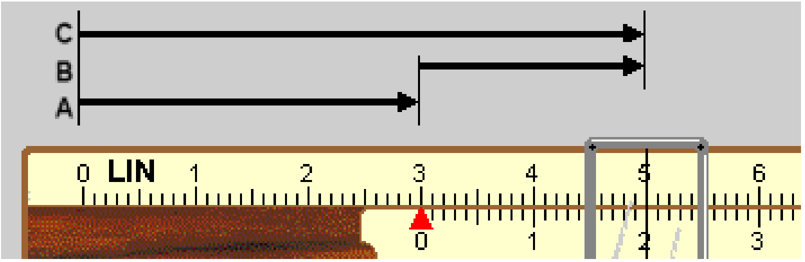
1 is	0
10	1
100	2
1000	3
10,000	4
100,000	5

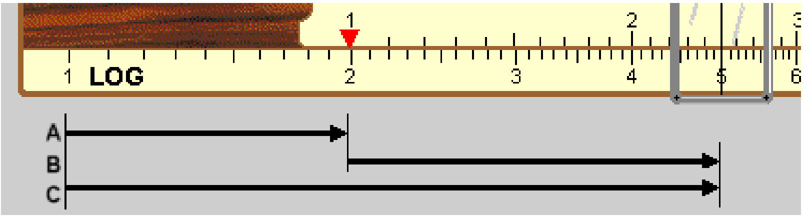
1 is	0.0
2	.301
3	.477
4	.602
5	.699
6	.778
7	.845
8	.903
9	.954
10	1.000

Exercise 5: creating a slide rule

- Meter sticks
- Log table
- Measure with a compass
- Multiply, multiply, multiply

Slide Rule

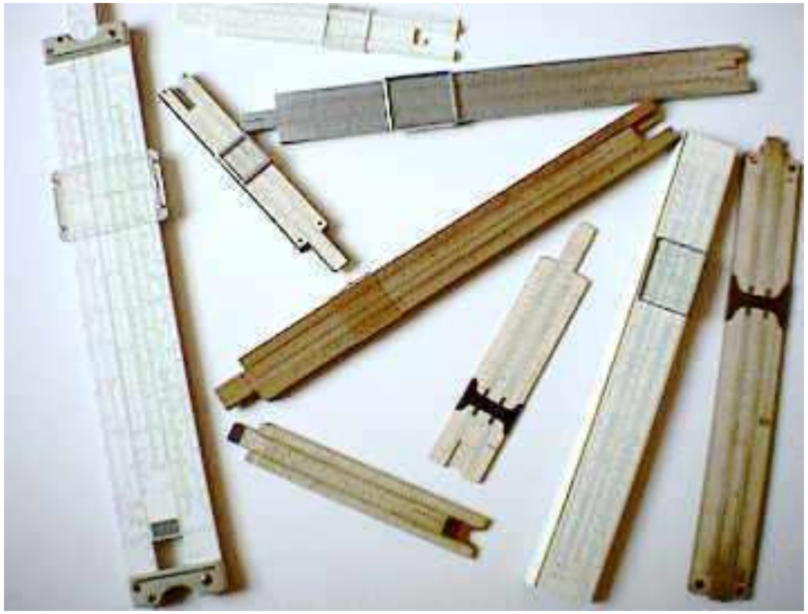


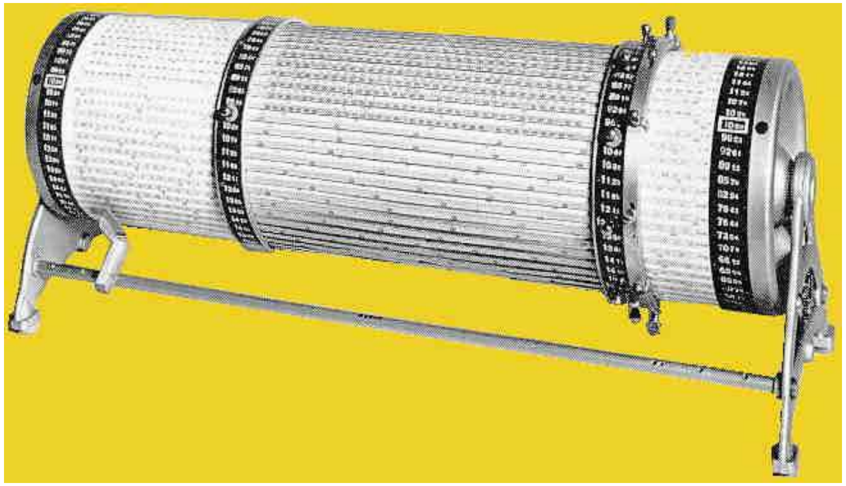


Exercise 6: Using the slide rule

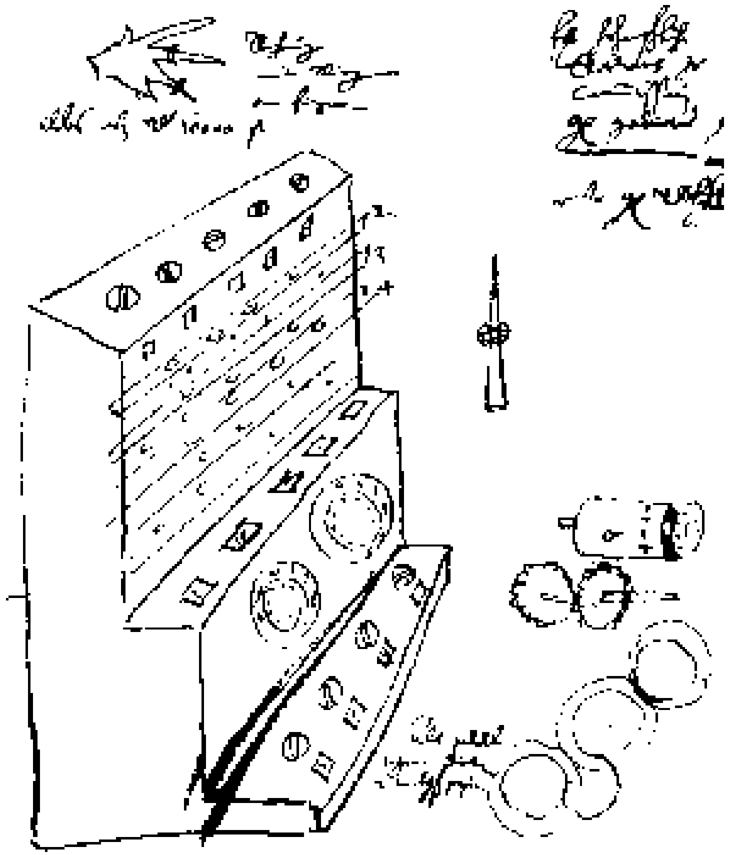
- Examine the slide rule
- Look at the scales: a, b, c, tan, etc.
- Multiply:
  - 2 times 2 (place left end of slide over 2
  - then move cursor to 2 on the slide
  - read down on the D scale
  - 8 times 8 (switch start point to right end)
  - 25 times 25 (hint: do 2.5 and then move decimal point in your head!)

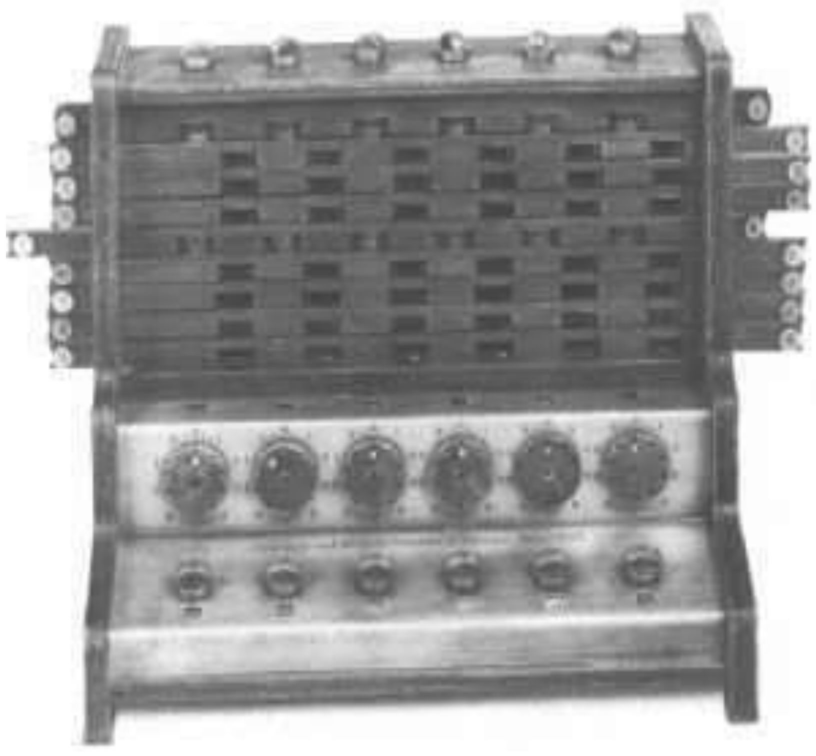
Manheim's *"Modern" Slide Rule* 1850

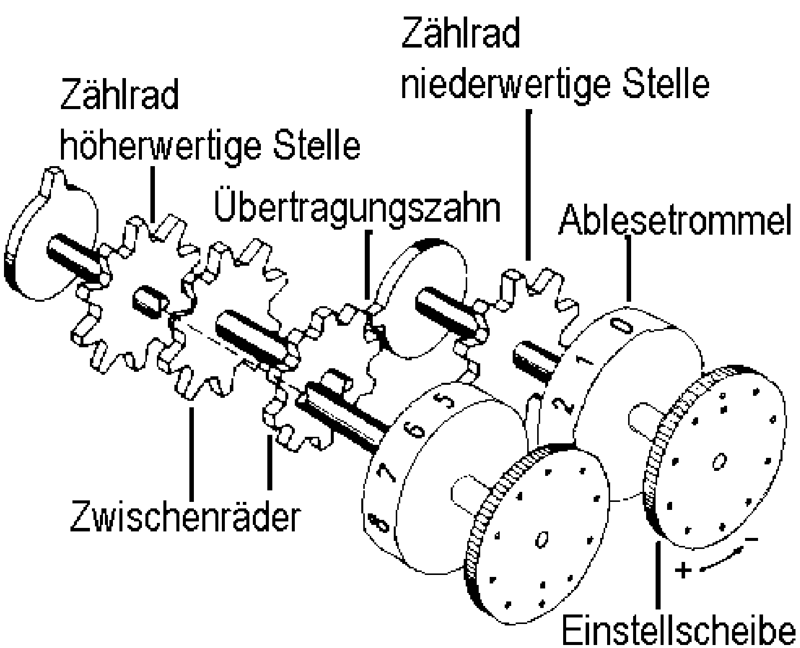




Wilhelm Schickard   (1592-1635)

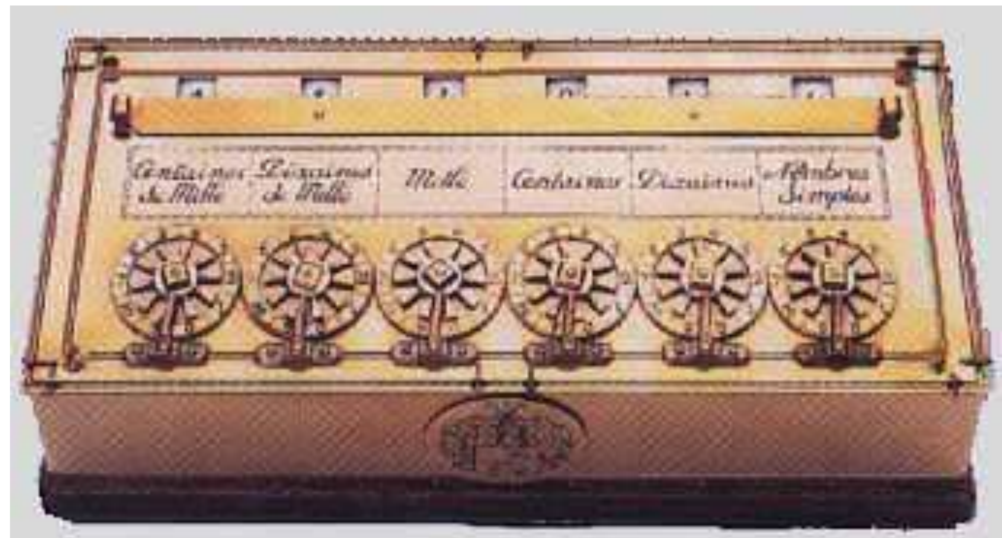






- Napier's bones on cylinders for multiplicand
- movable sliders to expose multiples (multiplier)
- gears with carry mechanism
- none exist; only known from drawing
- recreation by Dutch machinist @ 1992

Blaise Pascal (1623-1662)



The problem of calculation….

- Father was a tax collector in Rouen, France
- *Pascaline,* 1642

  (19 years old)
- Carry mechanism

  problem: carry propagation
- Made 50 Pascalines

Blaise Pascal

For after all what is man in nature? A nothing in relation to infinity, all in relation to nothing, a central point between nothing and all, and infinitely far from understanding either. The ends of things and their beginnings are impregnably concealed from him in an inpenetrable secret. He is equally incapable of seeing the nothingness out of which he was drawn and the infinite in which he is engulfed.

Ever forward, never backward….

- Pascaline could only do addition and NOT subtraction because the gears could only turn clockwise!
- Complimentary arithmetic: subtraction can be effected by adding the 9's compliment to the number.
- The compliment of 2 is 7, of 3 is 6, etc.

Complimentary Arithmetic

294736   294736

- 217485---------------->+ 782514

1077250

truncate leftmost position^

add one to rightmost position +1

077251

Check: 294736

- 217485

77251

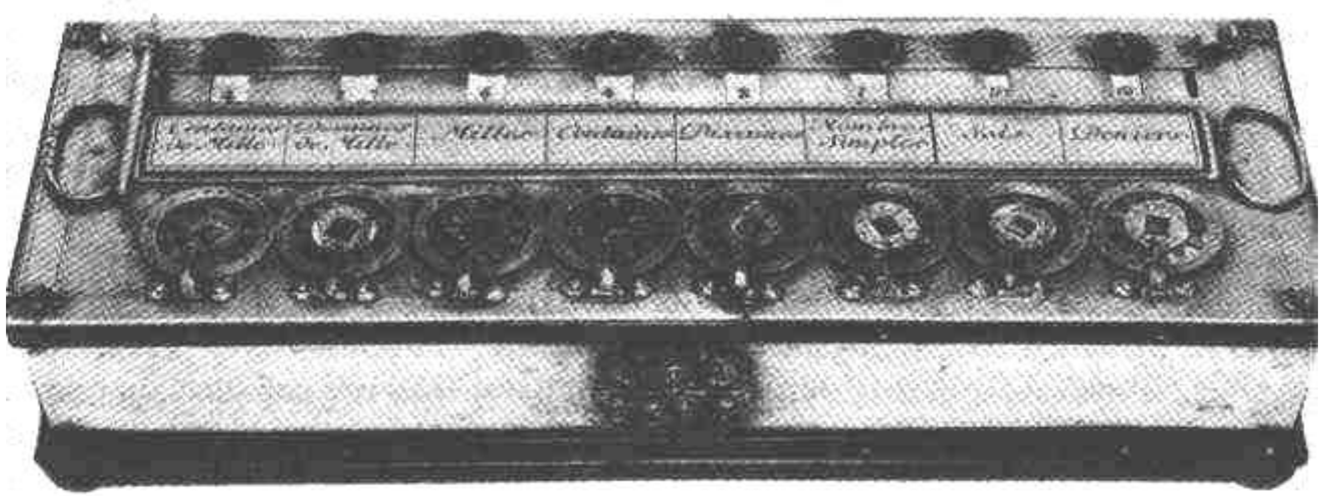
Exercise 7: Complimentary Arithmetic

$$\begin{array}{r} 6874 \\ \underline{-2846} \end{array}$$

$$\begin{array}{r} 485967 \\ \underline{-\ 34556} \end{array}$$

Be sure to check your work

Musee des Arts et Metiers



Gottfried Wilhelm Leibniz (1646-1716)

It is unworthy for excellent men to lose hours like slaves in the labour of calculation which could safely be relegated to anyone else if machines were used.

Leibniz's *Stepped Drum Calculator (1674)*

Sources

- Michael R. Williams, *A History of Computing Technology*, IEEE Computer Society Press, 2nd edition, 1997
- www.arithmeum.de, a museum for the history of calculation, in Bonn, Germany (from the collection of Proof. Bernhard Korte, University of Bonn)

Show and Tell

- Pebbles
- Chinese Swan Pan, Japanese Soroban
- *Jetons*
- Napier’s bones
- Arabic times table
- Schickard’s Calculator
- Pascal-like calculator from The Computer Museum
- Books of Logarithm tables
- Meter sticks and compass
- Slide rules

Laboratory

- 1- Using the Abacus
- 2- Finger Reckoning
- 3- Using logarithm tables
- 4- Gelosia Method
- 5- Using Meter sticks to create a slide rule
- 6- Using the slide rule
- 7- Complimentary Arithmetic