MATHEMATICA 101:
TECHNIQUES AND TIPS FOR YOUR AVERAGE MATH 118 STUDENT

Let's be real here, until you got to MATH 118 most of you have probably never seen or even heard of this new fan-dangled technology called Mathematica. Well don't worry. Like many of you, us past 118 students have felt the struggle. With plenty of confusion over what could have possibly gone wrong with input to make the program flip out and the simple pain of "how in the world do I do this???", we've finally decided it was time to stop the struggle. Welcome to Mathematica 101; your new best friend for the semester.

Let's get some formalities out of the way first. I will be showing you each of these techniques/tips/cheats using Mathematica 10.0 - Student Edition. In some older versions of the program the commands are not the same. However, Haverford has been so kind as to give us a free download from IITS here.

1. HOW TO MATHEMATICA

So let's start with some of the most basic input information about Mathematica and work our way up to the hard stuff. Baby steps, I know, but I promise we'll get there.

HELP

If this not **THE** most important function on Mathematica, I really don't know what is. At many times you will find that the program offers you a suggestion but you have literally *no idea* as to what they're trying to say. Good thing they also have a command for that as well. You can inquire about any string of text that resembles a command/function by starting your line with a ? and following it with what you're confused about.

```
In[65]:= ? Reduce

Reduce[expr, vars] reduces the statement expr
    by solving equations or inequalities for vars and eliminating quantifiers .
Reduce[expr, vars, dom] does the reduction over the domain dom. Common
    choices of dom are Reals, Integers , and Complexes .  >>
```

ENTERING A FUNCTION

You're probably thinking that as one of the most common commands to perform in Mathematica, it must be pretty easy right? Well... we'll see...

If you're trying to define a function which you would like to use with a short cut later on (because we're all lazy, let's face it), the image below shows what your inputs could look like.

```
In[22]:= F[x_] := x^2 + 6

In[23]:= f := x^2 + 6
```

The :=, or "set delayed" symbol is critical here as it is what keeps the set function for later computations like the one below.

```
In[14]:= F[x_] := x^3 + 2 x - 4

In[15]:= F[3]

Out[15]= 29
```

This clearly works with the first function notation which we input in Mathematica, however this does not work with our second form of notation. Although the “set delayed” symbol does save the function in the program’s memory to use later, the lowercase f represents an *expression* rather than a function. Therefore when we try to use this notation in the same way, we can see below that the evaluation for a specific value of x does not occur, and rather uses it as its own variable.

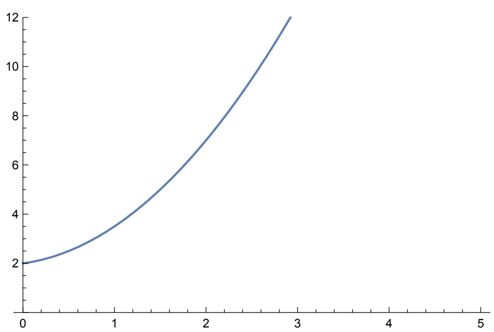
```
In[16]:= f := (x - 17 x^2) (x^3 + 4 x - 1)

In[18]:= f (6)
```

Out[18]= $6\left(x - 17\,x^2\right)\left(-1 + 4\,x + x^3\right)$

Many times you want to do more than one command with your function. You’re probably thinking, “Oh boy, I’m going to have to type 4 lines worth of commands to do something simple. Ew...” Well, you’re in luck! Turns out you can link multiple commands together by just using commas! An example of this is shown below where we give the command to plot a specific function **and** change the window.

```
In[13]:= Plot[{x^2 + 1 / 2 x + 2}, {x, 0, 5}, PlotRange → {0, 12}]
```

Out[13]=



Most functions which you will be using in regards to DDSs and DEs this semester willl involve the mulitplication of variables. In both the applications built by our lovely Haverford math professors and Mathematica alike, however, it is necessary to know that you **cannot** just smack 2 variables together and expect everything to be fine and dandy. Turns out, Mathematica recognizes this incorrect notation as a new variable instead of the multiplication of 2 variables. To avoid this issue and some inevitable freak out by the program, the multiplied variables need to be separated by a space *or* need to have an asterisk between them. As you can see in the example below, Mathematica does not solve for when xy are in the function because it no longer sees that x is its own independent variable. In the second equation it recognizes the space as multiplication and adjusts accordingly.

```
In[46]:=  Solve[{0 == xy + y}, x]

Out[46]=  { }

In[47]:=  Solve[{0 == x y + y}, x]

Out[47]=  {{x → -1}}
```

Something you should also take note of when entering your functions/expressions into Mathematica: when you have a variable at any time, the variable term will appear blue. This will occur whenever you have an *undefined* variable. When you have a *defined* variable it should appear teal.

```
          Sin[x]

In[110]:= e^x

Out[110]= e^x

In[111]:= e^x

Out[111]= e^x
```

As you can see in the above figure, when we have our undefined variable, even in a trigometric function, it remains blue. Then below that we can see the difference between blue e representing a variable first, and then black e representing the specific mathmatical number.

ENTERING SPECIFIC TERMS

As if things weren't already complicated, most commands and specific functions which are typed into Mathematica require you to enter them in very specific ways. Let's start with our trig functions first.

```
In[19]:= Sin[x]

         Cos[x]

         Tan[x]

         ArcTan[x]
```

When entering trig functions, you *must* enter them with a capital letter beginning the specific function which you're referencing, otherwise Mathematica will not recognize the function behind string of letters which you've typed hoping that it would work. After the trig function name, as you can see you need hard braces which contain your variable **or** your expression/function in curly braces.

Another simple function which you need to be careful with is a square root. To do so, you must use the order as shown below with your numerical value/variable/function in the hard braces (the function of course being within curly braces within the hard braces if necessary).

Now we can move onto special constants. In order to insert the symbols for these constants which you want to use either in functions or expressions, you must enter them in a specific manner.

In[20]:= `Sqrt[49]`

Out[20]= 7

In[21]:= `Sqrt[x]`

Out[21]= $\sqrt{x}$

These are terms like pi, e, i, and infinity. They each require specific key ordering. For e, the keys are esc-e-e-esc, for pi esc-p-esc, for i esc-i-i-esc, for ∞ esc-inf-esc, for an integral esc-int-esc, and for lambda esc-lambda-esc.There are endless more symbols and constants you can write which you may see on a drop down menu after typing esc-esc. Some are shown below.

$$e \quad \pi \quad i \quad \infty \quad \int \quad \lambda$$

EVALUATION

Now that we know the basics of entering in functions and specific symbols, we can go ahead and evaluate and solve our functions in different ways.

So why exactly are you using Mathematica in the first place? Probably to solve equations which are too complicated to do by hand right? So let's teach you how to do just that. When you have an equality, you need to solve by using the Solve function with hard brackets. Inside the brackets, we have our function within curly brackets. When typing in your function you have to make sure to have both sides of the equality present, each respectively on opposite sides of *double* equals signs. Outside the function brackets, should exist containing what variable/term you are solving for. An example of this is shown below.

In[49]:= `Solve[{0 == x^4 - 3 x^2 + 2 x}, x]`

Out[49]= $\{\{x \to -2\}, \{x \to 0\}, \{x \to 1\}, \{x \to 1\}\}$

In[50]:= `Solve[{x^2 == 3 x + 2}, x]`

Out[50]= $\left\{\left\{x \to \frac{1}{2}\left(3 - \sqrt{17}\right)\right\}, \left\{x \to \frac{1}{2}\left(3 + \sqrt{17}\right)\right\}\right\}$

In[51]:= `Solve[{y == x^2 + 6}, x]`

Out[51]= $\left\{\left\{x \to -\sqrt{-6 + y}\right\}, \left\{x \to \sqrt{-6 + y}\right\}\right\}$

We also like to evaluate functions and equations with limits. Limits also have interesting inputs and can sometimes be a little tricky. We use the command Limit with hard brakets. Inside those hard brackets, per usual we have our function inside curly brackets or our pre-defined function f. We also have our changing variable and what it is approaching as a member of the limit. An example where x is approaching ∞ is shown below.

```
In[106]:= Limit[{-x / ⅇ^x - 1 / ⅇ^x + 1}, x → Infinity]
Out[106]= {1}
```

DERIVATIVES

Now we can move onto derivatives. Really start to get into the calculus.

When taking the derivative of a function which we've already defined with your function f that was defined earlier with the "set delay" symbol. You can take the derivative simply by using the notation shown below, where you just have your f and respective variable within the derivative command D's hard braces separated by a comma. However, as you can see, if you're using a function which you have not defined yet you must use the curly braces to enter your new function into the braces.

```
In[35]:= f := 6 x^2 + 5 x - 13

In[36]:= D[f, x]
Out[36]= 5 + 12 x

In[37]:= Clear[x]

In[38]:= D[{x^2 + 3 x + 4}, x]
Out[38]= {3 + 2 x}
```

Note that the Clear command was used to clear any values of x which existed prior to then.

Now what if you want to take the derivative with respect to a different variable? This occurs occasionally when taking partial derivatives or the derivatives of autonomous DEs. This change from a derivative with respect to x is fairly simple. The notation is almost identical to that shown above, but the variable after the comma is that which respect is taken to. This slightly changed notation is shown below.

```
In[43]:= D[{x y + x^2 y - 3 y}, y]
```

Out[43]= $\{-3 + x + x^2\}$

As you've seen from previous calculus courses, it is possible to take the nth derivative of some function. In order to do this in Mathematica you can enter in your traditional derivative notation, but instead of just having the respectful variable after the comma, you have a set of curly brackets which contain our variable with which the derivative is taken with respect to, followed by a comma and the number of derivatives taken, n.

In[39]:=
```
f := 6 x^3 + 2 x^2 - x + 2
```

In[40]:=
```
D[f, {x, 2}]
```

Out[40]= $4 + 36 x$

In[41]:=
```
Clear[x]
```

In[42]:=
```
D[{3 x^6 + 4 x^2 + 3}, {x, 4}]
```

Out[42]= $\left\{1080 x^2\right\}$